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Visually-guided investigations of sub-structures in 3D Turing-like patterns

Turing's Reaction Diffusion

Turing proposed a reaction-diffusion model for skin coloring [1] which was subsequently discretized by Young [2] as a CA. We investigate the parameter space of the corresponding 3D models [3]. Starting from a random activation probability $\rho \in [0,1]$, each cell (x,y) has a status $s_t(x,y)$ given by:

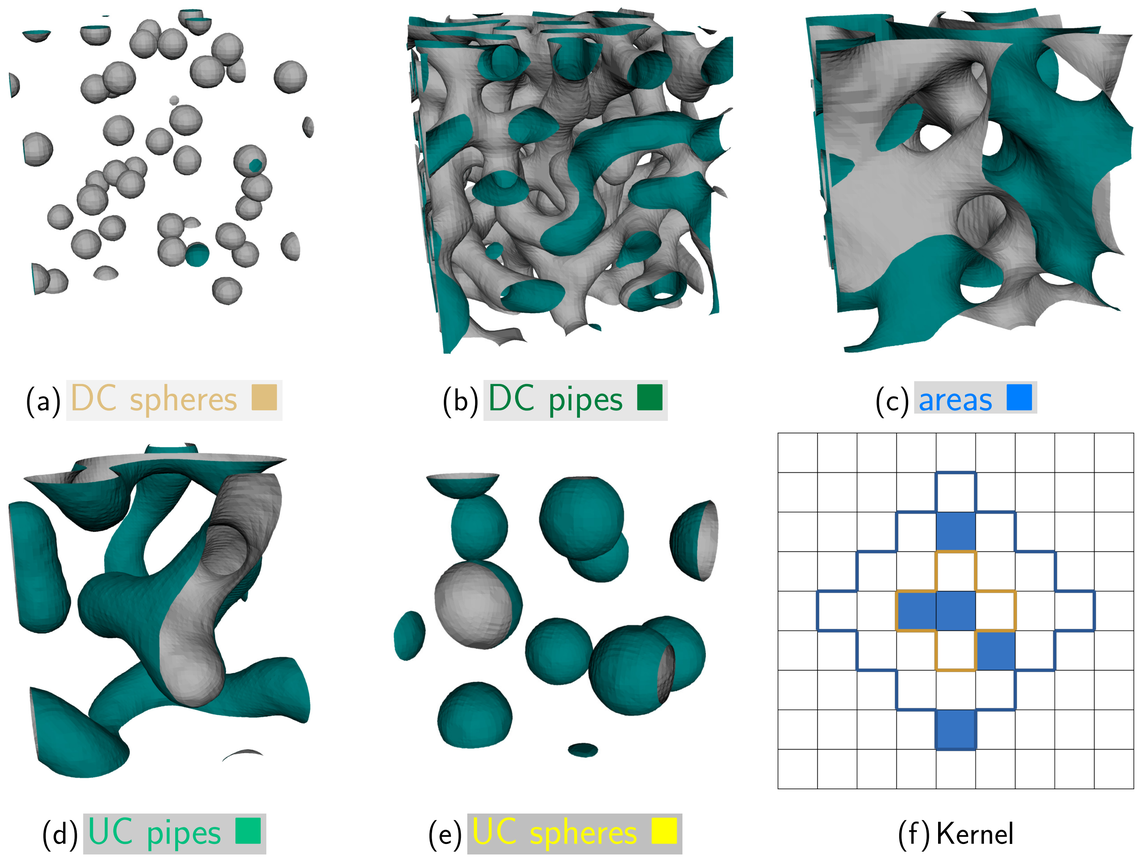
$$s_{t+1}(x,y) = \begin{cases} 1 & \sum_{(x',y')\in B_{R_2}(x,y)} \omega_{t,(x,y)}(x',y') > 0, \\ s_t(x,y) & \sum_{(x',y')\in B_{R_2}(x,y)} \omega_{t,(x,y)}(x',y') = 0, \\ 0 & \sum_{(x',y')\in B_{R_2}(x,y)} \omega_{t,(x,y)}(x',y') < 0, \end{cases}$$

where $B_{R_2}(x,y)$ is the ball of radius R_2 around (x,y) and

$$\omega_{t,(x,y)}(x',y') = \begin{cases} 0 & (x-x')^2+(y-y')^2 > R_2^2, \\ w_1 \cdot s_t(x',y') & (x-x')^2+(y-y')^2 < R_1^2, \\ w_2 \cdot s_t(x',y') & \text{otherwise.} \end{cases}$$

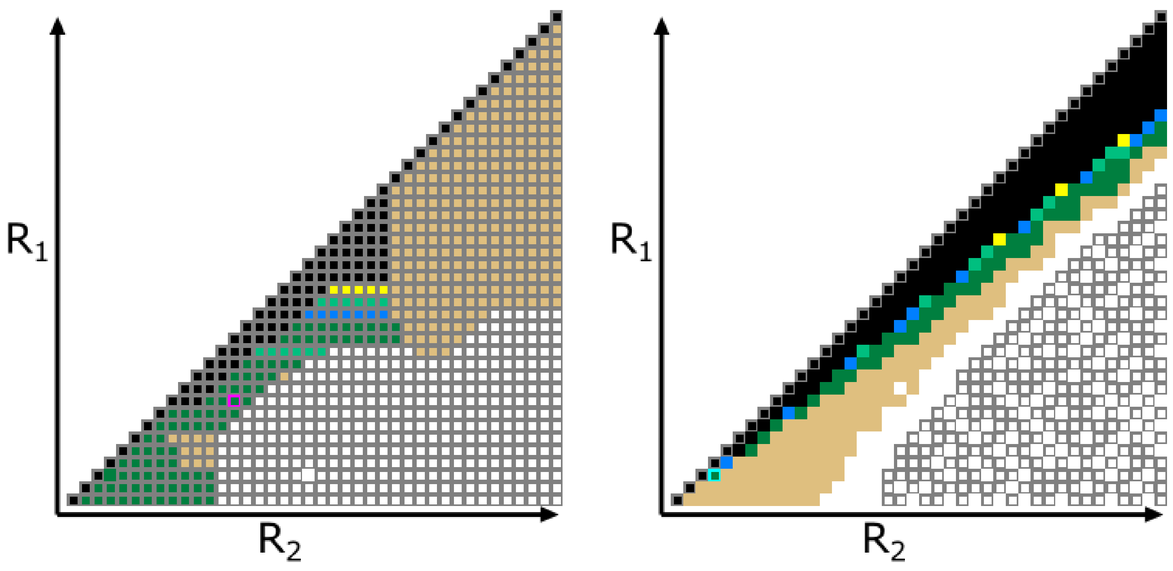
On a three-torus, this model creates a three-dimensional parameter space (ρ, R_1, R_2) with seven possible expressions. We want to understand their distribution.

Five non-trivial patterns and Kernel



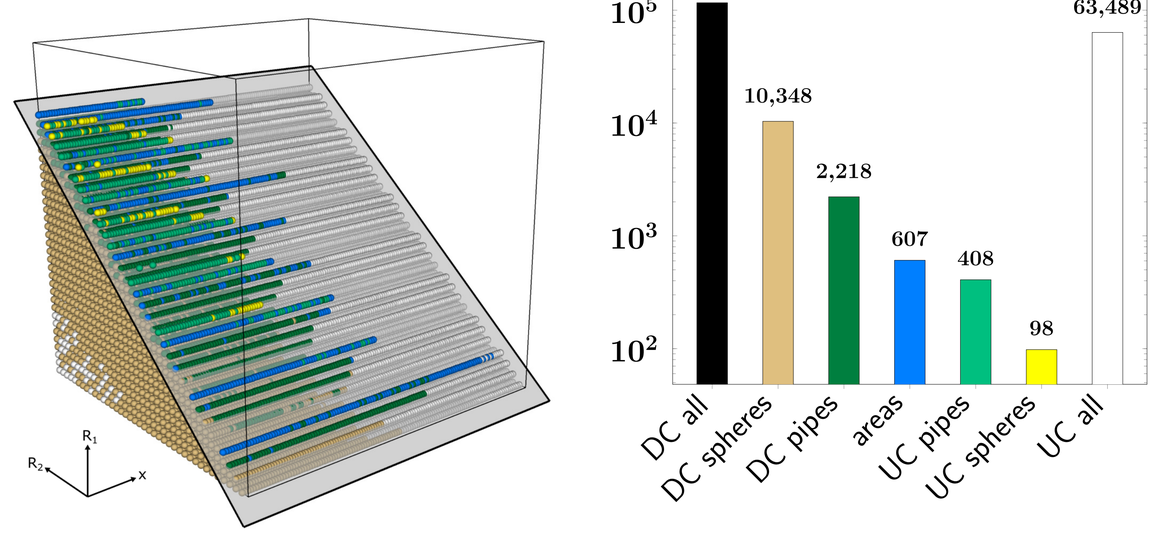
(a) DC spheres (b) DC pipes (c) areas (d) UC pipes (e) UC spheres (f) Kernel

2D Slices through the 3D Parameter Space for fixed ρ



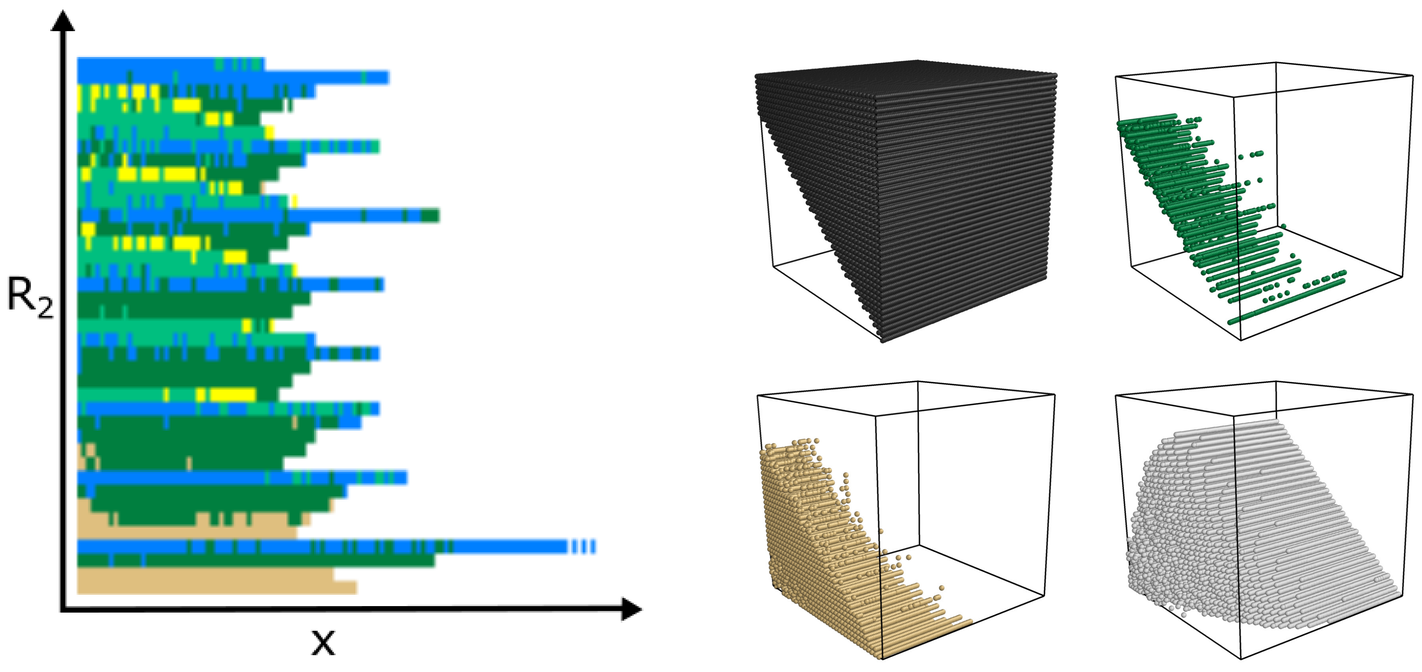
Each ρ specifies a slice through the three-dimensional parameter space. Screenshots from our program. Gray box indicates unexplored parameters with an estimate for most probable outcome.

3D Parameter Space and Pattern Histogram



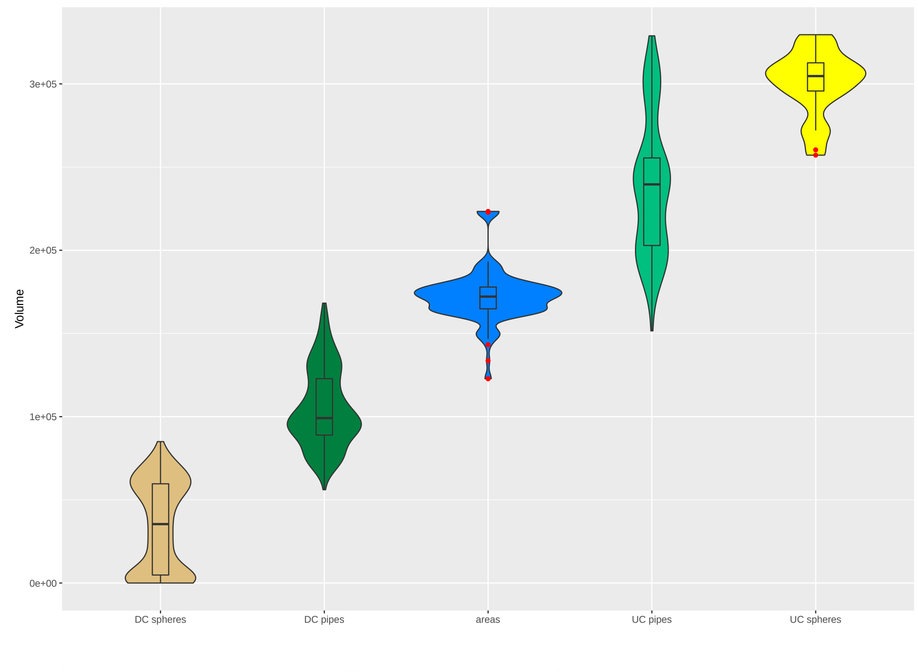
On the left the 3D parameter space with each parameter triplet colored according to its pattern. On the right a histogram on the occurrence of patterns.

Partition of the 3D Parameter Space



Partition of the three-dimensional parameter space. Left image shows the top layer with the sparsest patterns, four right images show those patters that exhibit volumetric extent.

Automatic Pattern Detection



Violin plot over the different types of sub-structures, indicating the volume that is taken by each sub-structure. Volume is a predictor for up to 90% of all patterns.

References

[1] Turing, A.M.. 1952. "The chemical basis of morphogenesis." Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences 237 (641):37–72.

[2] Young, D.A. 1984. "A local activator-inhibitor model of vertebrate skin patterns." Mathematical Biosciences 72 (1):51–58.

[3] Skrodzki, M., Reitebuch, U., and Zimmermann, E. 2020. "Experimental visually-guided investigation of sub-structures in three-dimensional Turing-like patterns". ArXiv, abs/2006.16676.